HLM

Hierarchical Linear Modeling

Lesson Three

Lesson Three Plan

I. Review of I and II

II Null Model

III R-Squared in HLM

IV Shrinkage and the Reliability of Estimates

The Model

Y_{ij} = 👓$_{0j}$ + 👓$_{1j}X_{1ij}$ + 👓$_{2j}X_{2ij}$ + 👓$_{3j}X_{3ij}$ + 📫$_{ij}$ within

👓$_{0j}$ = 💣$_{00}$ + 💣$_{01}Z_{1j}$ + 💣$_{02}Z_{2j}$ + U_{0j}

👓$_{1j}$ = 💣$_{10}$ + 💣$_{11}Z_{1j}$ + 💣$_{02}Z_{2j}$ + U_{1j} between

👓$_{2j}$ = 💣$_{20}$ + 💣$_{21}Z_{1j}$ + 💣$_{02}Z_{2j}$ + U_{2j}

👓$_{3j}$ = 💣$_{30}$ + 💣$_{30}Z_{1j}$ + 💣$_{02}Z_{2j}$ + U_{2j}

Model Adequacy

First fit a model without any group-level (i.e. Z) variables:

	Effect Significant? (T-test)	Varies Significantly Across Groups (X^2 - test)	
a	No	No	Consider dropping from the model but consider its role in analysis
b	No	Yes	Retain as a random factor
c	Yes	No	Retain as a fixed factor
d	Yes	Yes	Retain as a random factor

In case b and d, you can then specify and test between-group models.

Null Model

The null model is useful at the outset to determine what proportion of the variance lies between groups.

$Y_{ij} =$ 👓$_{0j}$ + 📪$_{ij}$ (1) 📪$_{i}$ 📫 NID(0,☞^{2}) within

👓$_{0j}$ = 💣$_{00}$ + U_{0j} (2) U_{0j} 📫 NID(0,☺) between

Substituting (2) into (1) yields:

$Y_{ij} =$ 💣$_{00}$ + U_{0j} + 📪$_{ij}$

💣$_{00}$ is the grand mean

U_{0j} is the group's deviation from the mean

📪$_{ij}$ is the individual's deviation from the mean.

Notice that the variance of the outcome is:

Var(Y_{ij}) = Var(💣$_{00}$ + U_{0j} + 📪$_{ij}$) = ☺ + ☞^{2}

This is equivalent to a one-way ANOVA model with random effects.

! produces point estimate and confidence interval for 💣$_{00}$.

! gives information about outcome variability at each level.

! allows you to calculate the intra-class correlation coefficient:

👎 = ☺ / (☺ + ☞^{2})

R-Squared

In OLS regression we can think of R-squared as the proportion of variance explained.

For example, if we started with a null model:

$$Y_i = \beta_0 + \varepsilon_i$$

Shrinkage and the Reliability of Estimates

see Raudenbush & Bryk (1986)

Within-Group Model: $Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \varepsilon_{ij}$

$N(0,\sigma^2)$

we have j such equations, one for each group.

Between-Group Model: Assume no knowledge of group-level factors)

$\beta_{0j} = \gamma_{00} + U_{0j}$

$N(0,\tau_0)$

τ_0 is the "true" parameter variance of β_{0j}

$\beta_{1j} = \gamma_{10} + U_{1j}$

$N(0,\tau_1)$

- if we centred the data within schools we could have:

👓$_{1j}$ = 💣$_{10}$ + U_{1j}

average slope   N(0,☺$_1$)

T_1 is the “true” parameter variance of B_{1j} (i.e., variance of true outcome/SES slopes across schools

- if we used OLS for each of the j within-group regressions, we would obtain "observed" slopes :

Observed = True + Sampling Error

👓$_{1j}$ = 👓$_{1j}$ + e_j

N(0,V_j)

where V_j = Variance (👓$_{1j}$|👓$_j$) = ☞^{2}/🕯x^2

- taking variances we get :

Var(👓$_{0j}$) = Var(👓$_j$) + Var(e_j)

= ☺ + V_j

parameter variance   sampling variance

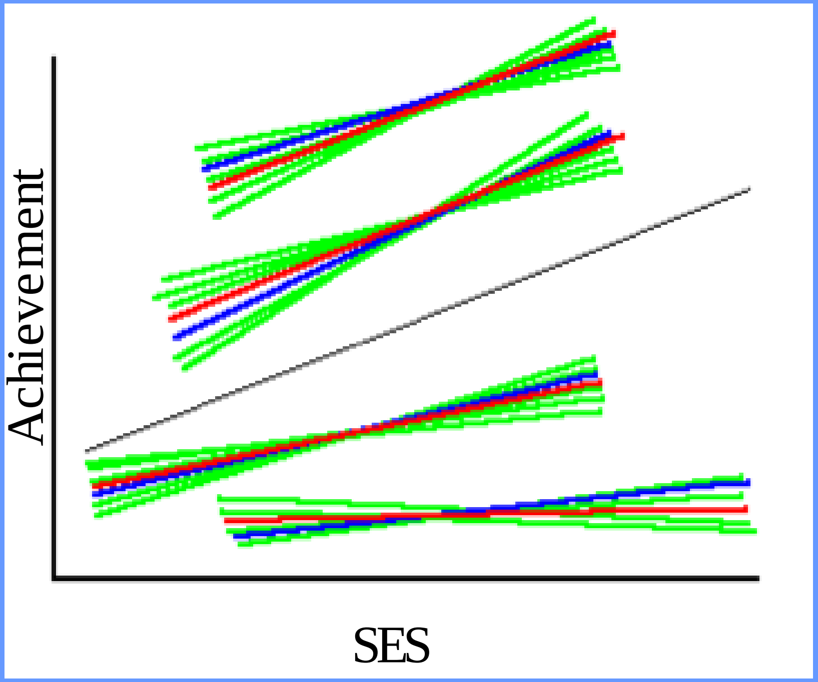
- Under Empirical Bayes :

👓$_j$* = w_j👓$_j$ + (1-w_j)💣$_{10}$*

This is a weighted sum of the OLS slope and the mean slope for the population of schools

where $\Phi_{10}^{*} = \dfrac{\Sigma w_j \beta_{1j}}{\Sigma w_j}$ and $w_j = \dfrac{T}{T + V_j}$

Like a classical test theory reliability



Φ_{10}

Alternate Slope

True slope 👓$_{1j}$ [Their “true” variance is T]

Observed slope 👓$_{1j}$ [Variance of observed slopes around their true slopes is V_j]

Empirical Bayes slopes [A weighted average of the average slope and the average slope. 💣$_{10}$]

The extent of "shrinkage" depends on the ratio $T/(T+V_j)$. When our within-group estimates are precise (large n_j and heterogeneous x), V_j is small, so the ratio $w_j = T/T(T + V_j)$ approaches 1.0. Thus there is little shrinkage. But if V_j is large relative to T, W_j approaches 0. Then each 👓$_{1j}$ is shrunk towards 💣$_{10}$.